Name-Brand vs. Off-Brand: A Twist on Taste Testing for a Mathematical Statistics Course

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Outline

Dataset and Story

The Mathematical Statistics Course

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- Topics include probability, properties of random samples, estimation, and inference via maximum likelihood.
- Exposes students to the theory underlying many methods encountered in other courses.
- It is really easy to divorce this course from the pedagogical tools we use in an Introductory Statistics course.

Background for Students

Name-Brand vs. Off-Brand: Can We *Really* Distinguish Between the Two?

Activity and Assignment

Data Collection

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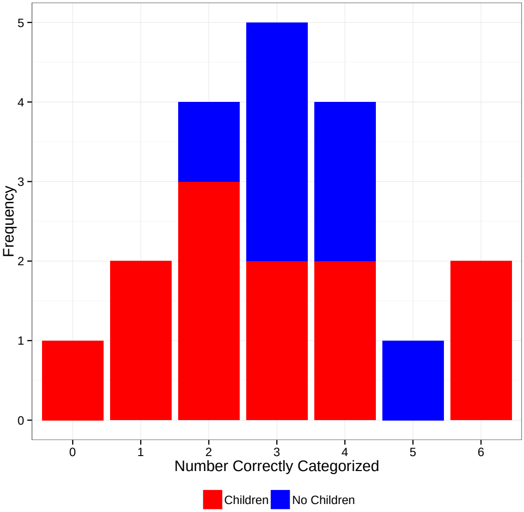
- What is the benefit of blinding and how might it be implemented?
- How will randomization be utilized in this study?
- What variables should be recorded in this study?

The Data


5
4
3
2
1
0
Frequency
0
1
2
3
4
5
6
Number Correctly Categorized
Children
No Children

The Assignment

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- Describe the distribution of the probability of actually discerning between the name-brand and off-brand products.
- Account for the fact that a correct response could be due to a true discernment or a lucky guess.
- Construct a poster-presentation summarizing your analysis and results.

Analysis

Model for Data-Generating Process

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- Assume $P_i \overset{IID}{\sim} Beta(\alpha, \beta)$.
- Then, the number correct is $Y_i \mid C_i \sim Bin(6, C_i)$.

Probabilistic Model

- Letting $f(y \mid \alpha, \beta)$ represent the marginal density of Y_i, then we have that

$$\ell(\alpha, \beta \mid y) = \sum_{i=1}^{n} \log\left[f\left(y_i \mid \alpha, \beta\right)\right]$$

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- Determining the marginal density is a good exercise in its own right (Casella and Berger [Statistical Inference (pg 197)]).
- Students are expected to show that

$$f(y \mid \alpha, \beta) = \frac{\binom{m}{y}\Gamma(\alpha+\beta)}{2^m\Gamma(\alpha)\Gamma(\beta)} \sum_{k=0}^{y} \binom{y}{k} \frac{\Gamma(\alpha+k)\Gamma(\beta+m-y)}{\Gamma(\alpha+\beta+m-y+k)}$$

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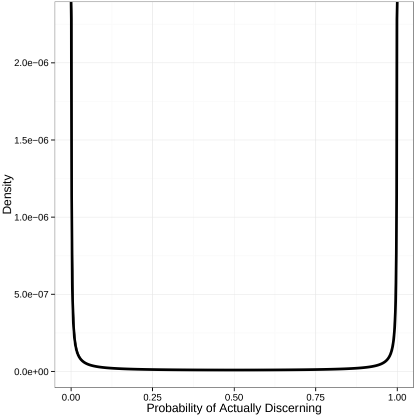
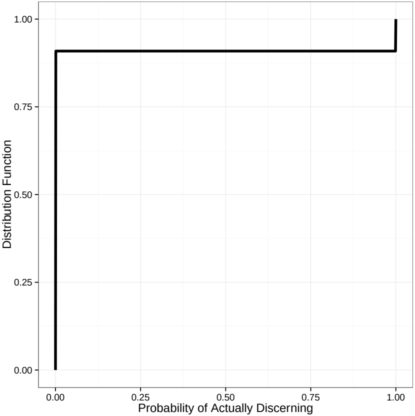
- Students constructed `R` programs to compute the estimates using the data.
- Due to numerical instability, care had to be given to the optimization routine chosen and the coding of the likelihood.

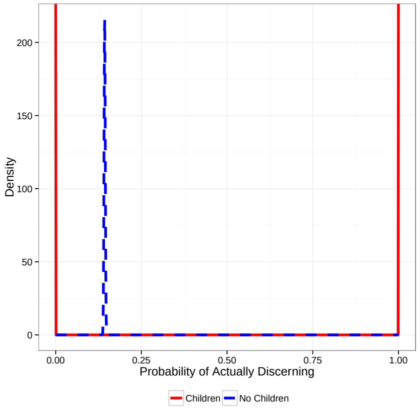
Results

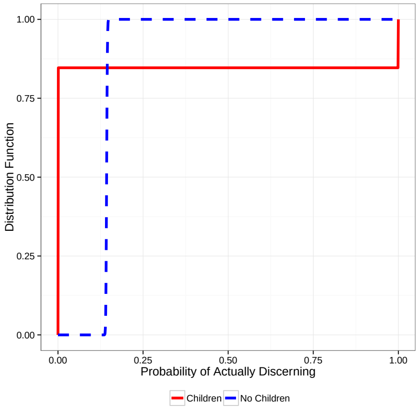
Results for Primary Question





Results Comparing Those with and without Children





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- Activity ties together several topics used throughout the course, and is a nice capstone to maximum likelihood.
- Lends itself well to a short poster presentation.

References

- Casella G and Berger RL.
  Statistical Inference (2nd ed, pg 197).
  Pacific Grove, CA: Thomson Learning, 2002.
- Morrison DG.
  A Probability Model for Forced Binary Choices.
  The American Statistician, 32(1):23-25, 1978.

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